Circles

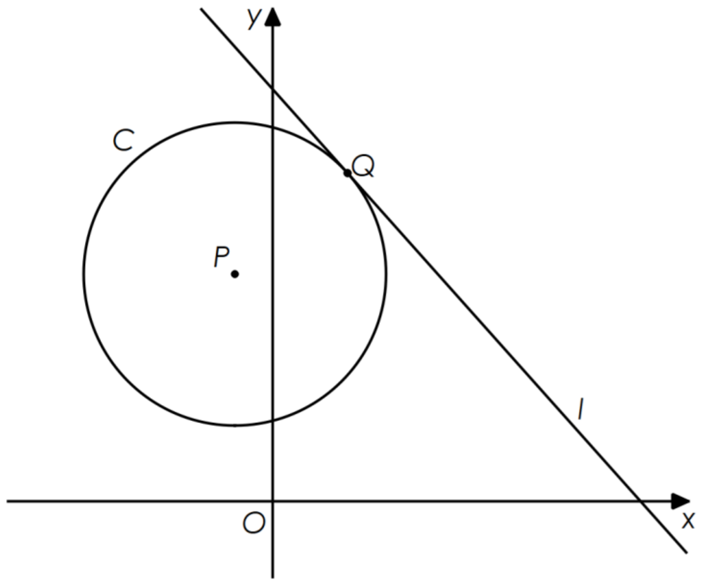

Figure 1

Figure 1 shows a sketch of the circle C.

- the point $P(-1, k + 9)$ is the centre of C
- the point $Q(2, k^2 - 3k + 3)$ lies on C
- k is a positive constant
- the line l is a tangent to C at Q

Given that the gradient of l is $-\frac{1}{2}$

a. show that

$$k^2 - 4k - 12 = 0$$

(4 marks)

b. Hence find an equation for C

(4 marks)

a. Find an expression for the gradient of PQ using the coordinates of P and Q:

$$\frac{k^2 - 3k + 3 - k - 9}{2 - -1} = \frac{k^2 - 4k - 6}{3}$$

1 mark

The gradient of the tangent to C is $-\frac{1}{2} \Rightarrow$ the gradient of the normal is 2

1 mark

As PQ is a normal to the line l:

$$\frac{k^2 - 4k - 6}{3} = 2$$

$$k^2 - 4k - 6 = 6$$

1 mark

$$k^2 - 4k - 12 = 0$$

1 mark

b. $k^2 - 4k - 12 = 0 \Rightarrow (k + 2)(k - 6) = 0$

$$k = -2 \text{ or } k = 6$$

As k is a positive constant, $k = 6$

1 mark

The point P has coordinates $(-1, 6 + 9) = (-1, 15)$

1 mark

The point Q has coordinates $(2, 6^2 - 3 \times 6 + 3) = (2, 21)$

The length $PQ = \sqrt{(2 - -1)^2 + (21 - 15)^2}$

1 mark

$$= \sqrt{3^2 + 6^2}$$
$$= \sqrt{9 + 36}$$
$$= \sqrt{45}$$

So the equation of the circle C is:

$$(x + 1)^2 + (y - 15)^2 = 45$$

1 mark