

## Vectors

[In this question the unit vector  $\mathbf{i}$  and  $\mathbf{j}$  are due east and due north respectively.]

A stone slides horizontally across ice.

Initially the stone is at the point  $A(-14\mathbf{i} + 48\mathbf{j})\text{m}$  relative to a fixed point  $O$ .

After 30 seconds the stone is at the point  $B(7\mathbf{i} - 24\mathbf{j})\text{m}$  relative to the fixed point  $O$ .

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

a. prove that the stone passes through  $O$ ,

**(2 marks)**

b. calculate the speed of the stone.

**(3 marks)**

a.  $(-14\mathbf{i} + 48\mathbf{j}) = -2 \times (7\mathbf{i} - 24\mathbf{j}) \Rightarrow \overrightarrow{OA} = -2 \times \overrightarrow{OB}$

1 mark

$\overrightarrow{AO}$  is parallel to  $\overrightarrow{OB}$ , therefore points  $A$ ,  $O$  and  $B$  lie on the same straight line and so the stone passes through the origin.

1 mark

b. First, find the distance from  $A$  to  $B$

$$\begin{aligned} |AB| &= \sqrt{(7 - (-14))^2 + (-24 - 48)^2} \\ &= \sqrt{21^2 + 72^2} \\ &= \sqrt{5625} \\ &= 75 \text{ m} \end{aligned}$$

1 mark

Speed = distance  $\div$  time

$$= 75 \div 30$$

1 mark

$$= 2.5 \text{ ms}^{-1}$$

1 mark