

## Cubic Equations

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

a. Using algebra, find all solutions of the equation

$$4x^3 - 5x^2 - 6x = 0$$

(3 marks)

b. Hence find all real solutions of

$$4(y-5)^6 - 5(y-5)^4 - 6(y-5) = 0$$

(3 marks)

a. Take  $x$  out as a common factor:

$$4x^3 - 5x^2 - 6x = 0 \quad \Rightarrow \quad x(4x^2 - 5x - 6) = 0$$

1 mark

Then factorise the quadratic:

$$4x^3 - 5x^2 - 6x = 0 \quad \Rightarrow \quad x(4x+3)(x-2) = 0$$

1 mark

So,  $x = 0$  or  $(4x+3) = 0$  or  $(x-2) = 0$ , leading to:

$$x = 0, x = -\frac{3}{4}, x = 2$$

1 mark

b. Using  $x = (y-5)^2$ ,  $4(y-5)^6 - 5(y-5)^4 - 6(y-5) = 0$  can be written  $4x^3 - 5x^2 - 6x = 0$ , so  $(y-5)^2 = 0$  or  $(y-5)^2 = 5$ . Can ignore the negative solution for  $x$  as  $(y-5)^2$  must be positive.

1 mark

$$(y-5)^2 = 0 \Rightarrow y-5 = 0 \\ y = 5$$

1 mark

$$(y-5)^2 = 5 \Rightarrow y-5 = \pm\sqrt{5} \\ y = 5 \pm \sqrt{5}$$

1 mark