Proof

Statement 1: $p^3 + q^3$ is never a multiple of 3

a. Show, by means of a counter example, that Statement 1 is **not** true.

(1 mark)

Statement 2: When p and q are consecutive **odd** numbers $p^3 + q^3$ is a multiple of 4

b. Prove, using algebra, that Statement 2 is true.

(4 marks)

a. Choose values for p and q that are both multiples of 3, for example $p = 6$ and $q = 9$.

This gives $p^3 + q^3 = 6^3 + 9^3$

$= 216 + 729$

$= 945$

$= 3 \times 315$ which shows that it is a multiple of 3

1 mark

b. Two consecutive odd numbers can be represented by $2n + 1$ and $2n + 3$

1 mark

$(2n+1)^3 = 8n^3 + 12n^2 + 6n + 1$ and $(2n+3)^3 = 8n^3 + 36n^2 + 54n + 27$

1 mark

$(2n+1)^3 + (2n+3)^3 = 8n^3 + 12n^2 + 6n + 1 + 8n^3 + 36n^2 + 54n + 27$

$= 16n^3 + 48n^2 + 60n + 28$

1 mark

$= 4(4n^3 + 12n^2 + 15n + 7)$ which is a multiple of 4

1 mark