**Binomial Expansion**

$$g(x) = (3 + ax)^6 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of g(x) is 2500x³

a.  find the value of a.

**(4 marks)**

Using this value of a,

b.  find the constant term in the expansion of

$$\left(1 + \frac{1}{x^2}\right)(3 + ax)^6$$

**(3 marks)**

a.   Begin by expanding $(3 + ax)^6$ – only need the coefficient of the $x^3$ term though:

1 mark

$$^6C_3 \; 3^3 \; a^3 = 20 \times 27 \times a^3$$

1 mark

$$= 540a^3$$

This then gives:

$$540a^3 = 2500$$

$$a^3 = \frac{2500}{540}$$

$$a^3 = \frac{125}{271}$$

1 mark

$$a = \frac{5}{3}$$

1 mark

b.   The constant term of the product needs the constant term and the coefficient of the $x^2$ term from the expansion of $(3 + ax)^6$

$$3^6 \text{ and } ^6C_2 \; 3^4 \; a^2$$

1 mark

Adding them gives:

$$3^6 + {}^6C_2 \; 3^4 \; a^2 = 729 + 15 \times 81 \times \frac{25}{9}$$

$$= 729 + 3375$$

1 mark

$$= 4104$$

1 mark