Trigonometry

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

a. Show that

$$\frac{1}{\cos\theta} - \tan\theta \equiv \frac{\cos\theta}{1+\sin\theta} \qquad \theta \neq (2n-1)90° \qquad n \in \mathbb{Z}$$

(3 marks)

Given that $\cos 5x \neq 0$

b. solve for $0 < x < 90°$

$$\frac{1}{\cos 5x} - \tan 5x = 2\cos 5x$$

giving your answers to one decimal place.

(5 marks)

a. Begin by writing the LHS as a single fraction

$$\frac{1}{\cos\theta} - \tan\theta = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

1 mark

Multiply the numerator and denominator by $1 + \sin\theta$

$$\frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1+\sin\theta} = \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

1 mark

Cancelling $\cos\theta$ gives:

$$\frac{\cos\theta}{1+\sin\theta}$$

1 mark

b. Using what was done in part a

$$\frac{1}{\cos 5x} - \tan 5x = \frac{\cos 5x}{1+\sin 5x} \qquad \text{and so} \qquad \frac{\cos 5x}{1+\sin 5x} = 2\cos 5x$$

1 mark

$$\cos 5x = 2\cos 5x(1+\sin 5x)$$

$$\cos 5x = 2\cos 5x + 2\cos 5x\sin 5x$$

$$2\cos 5x\sin 5x + \cos 5x = 0$$

$$\cos 5x(2\sin 5x + 1) = 0$$

1 mark

$$\cos 5x = 0 \text{ or } \sin 5x = -\frac{1}{2}$$

1 mark

In the region $0 < 5x < 450°$, $5x = 90°$, $210°$ and $330°$, so $x = 18°$, $42°$ and $66°$

1 mark for any two, 2 marks for all three