Exponentials and Logarithms 1

The height, h metres, of a plant, t years after it was first measured, is modelled by the equation

$$h = 1.7 - 0.9e^{-0.1t} \qquad t \in \mathbb{R} \quad t \geq 0$$

Using the model,

a. find the height of the plant when it was first measured,

(2 marks)

b. show that, exactly 2 years after it was first measured, the plant was growing at approximately 7.4 cm per year.

(3 marks)

According to the model, there is a limit to the height to which this plant can grow.

c. Deduce the value of this limit.

(1 mark)

a. When the plant is first measured, $t = 0$, so

$$h = 1.7 - 0.9e^{0}$$

1 mark

$$h = 1.7 - 0.9$$
$$= 0.8 \text{ m or } 80 \text{ cm}$$

1 mark

b. The rate at which the plant is growing is obtained by differentiating the expression for h with respect to time.

$$\frac{dh}{dt} = 0.09e^{-0.1t}$$

1 mark

To find the rate of growth exactly 2 years after the plant was first measured, substitute $t = 2$ into the differentiated function.

$$0.09e^{-0.1 \times 2} = 0.07368...$$

1 mark

So the rate of growth is 0.074 metres per year (7.4 cm per year).

1 mark

c. As t gets bigger, $e^{-0.1t}$ tends toward zero. This means that h tends toward $1.7 - 0.9 \times 0$.

So the limit to the height to which this plant can grow is 1.7 m

1 mark