

AS Core Pure Mathematics Vol 2 Set 29

## 2. Vectors 2

The line I has equation

$$\frac{x-4}{2} = \frac{y-3}{-1} = \frac{z+8}{3}$$

The plane  $\Pi$  has equation

**r**.(2**i** + **j** − **k**) = 12

Determine whether the line *l* intersects  $\Pi$  at a single point, or lies in  $\Pi$  or is parallel to  $\Pi$  without intersecting it.

(5 marks)

Question	Scheme	Marks
	$(4, 16), (1, 19) \Rightarrow 16 = a(4)^2 + b, 19 = a(1)^2 + b \Rightarrow a =, b =$	M1
1a	$a = -\frac{1}{5}, b = \frac{96}{5}$	A1
	$\pi \times 4^2 \times 16$ and $\pi \times 1^2 \times 6$	B1
	$\pi \int x^2 dy = \pi \int (96 - 5y) dy$	B1ft
	$=\pi \int_{19}^{19} (96-5y) dy$	M1
lb	$=\pi \left[96y - \frac{5y^2}{2}\right]^{19}$	M1 A1
	$V = 262\pi + \pi \left[ 96(19) - \frac{5(19)^2}{2} - \left( 96(16) - \frac{5(16)^2}{2} \right) \right]$	ddM1
	$V = 287.5\pi \approx 903 \text{ cm}^3$	A1
lc	<ul> <li>Any one ot:</li> <li>the measurements may not be accurate</li> <li>the equation of the curve may not be a suitable model</li> <li>the bottom of the bottle may not be flat</li> <li>the thickness of the glass may not have been considered</li> <li>the glass may not be smooth</li> </ul> This part asks for a limitation of the model so their answer must refer to: <ul> <li>the measuring of the dimensions</li> <li>the model used for the curve</li> <li>the simplified model (the thickness of glass, the simplified shape, smoothness of the glass etc.)</li></ul>	B1
1d	<ul> <li>There are 2 criteria for this mark:</li> <li>a comparison of their value to 750 eg larger, smaller, about the same or a difference demonstrated eg 903 – 750 = but not just a percentage error or just a difference with no calculation</li> <li>a conclusion that is consistent with their values eg this is not a good model, this is a good model etc.</li> <li>If they reach an answer that is less than 750, they need to conclude that it is not a good model</li> <li>If they reach an answer that is greater than 750 then look for a sensible comment that is consistent with their value</li> </ul>	B1ft
	$(\mathbf{r} = ) \begin{pmatrix} 4+2\lambda \\ 3-\lambda \\ 0+2\lambda \end{pmatrix} \mathbf{or} \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \text{ (oe)}$	Ml
2	$ \begin{array}{c} (-8+3\lambda) & (-8) & (3) \\ \hline \text{So meet if} \\ \begin{pmatrix} 4+2\lambda \\ 3-\lambda \\ -8+3\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 12 \Rightarrow (4+2\lambda) \times 2 + (3-\lambda) \times 1 + (-8+3\lambda) \times -1 = 12 \end{array} $	M1 A1
	$\Rightarrow 0\lambda + 19 = 12 \Rightarrow 19 = 12$ a contradiction so no intersection	A1ft
	Honco Lic parallel to II but not in it	Ales

# AS Core Pure Mathematics Mark Scheme Vol 2 Set 29

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oe	or equivalent (and appropriate)

## 1. The *t*-Formulae 3

i. Use the substitution  $t = tan\left(\frac{x}{2}\right)$  to prove that

$$\cot x + \tan\left(\frac{x}{2}\right) = \operatorname{cosec} x \qquad x \neq n\pi, n \in \mathbb{Z}$$

(2 marks)

ii.

An engineer models the vertical height above the ground of the tip of one blade of a wind turbine, shown in Figure 1. The ground is assumed to be horizontal.

Figure 1

H metres

The vertical height of the tip of the blade above the ground, H metres, at time x seconds after the wind turbine has reached its constant operating speed, is modelled by the equation

 $H = 80 - 30\cos(90x)^{\circ} - 60\sin(90x)^{\circ}$ 

a. Show that H = 50 when x = 0

Using the substitution  $t = \tan (45x)^\circ$ 

b. show that equation (I) can be rewritten as

$$H = \frac{110t^2 - 120t + 50}{1 + t^2}$$

c. Hence find, according to the model, the value of x when the tip of the blade is 90m above the ground for the first time after the wind turbine has reached its constant operating speed.

(5 marks)

(3 marks)

# (I)

(1 mark)



Figure 2 shows a sketch of part of the rectangular hyperbola H with equation

$$xy = c^2 \qquad x > 0$$

where c is a positive constant.

The point 
$$P\left(\operatorname{ct}, \frac{c}{t}\right)$$
 lies on H

The line I is the tangent to H at the point P.

The line I crosses the x-axis at the point A and crosses the y-axis at the point B.

The region R, shown shaded in Figure 2, is bounded by the x-axis, the y-axis and the line I.

Given that the length OB is four times the length of OA, where O is the origin, and that the area of R is 72, find the exact coordinates of the point P.

(10 marks)

Question	Scheme	Mark
	$\cot x + \tan\left(\frac{x}{2}\right) = \frac{1-t^2}{2t} + t$	M1
	$\frac{1-t^2}{2t} + t = \frac{1-t^2+2t^2}{2t}$	
1i	$=\frac{1+t^2}{2}$	A 1
	27	AI
	$=\frac{1}{\sin x}$	
	= cosec x	
1iia	$x = 0 \Rightarrow H = 80 - 30\cos(0) - 60\sin(0) = 80 - 30 = 50$	B1
	$H = 80 - 30\cos(90x)^{\circ} - 60\sin(90x)^{\circ} = 80 - 30\left(\frac{1 - t^{2}}{1 + t^{2}}\right) - 60\left(\frac{2t}{1 + t^{2}}\right)$	M1
1iib	$=\frac{80+80t^2-30+30t^2-120t}{2}$	M1
	$\frac{1+t^2}{1+t^2}$	
	$= \frac{1107 - 1207 + 50}{1 + t^2}$ from correct working	A1
-18	$\frac{110t^2 - 120t + 50}{1 + t^2} = 90 \implies 110t^2 - 120t + 50 = 90 + 90t^2$	М1
	$20t^2 - 120t - 40 = 0$	A1
1iic	$t = 3\pm\sqrt{11} \Rightarrow 45x = \arctan(3+\sqrt{11}) \text{ or } 45x = \arctan(3-\sqrt{11})$	M1
	$90x = \arctan(3 + \sqrt{11}) = 81.00 \Rightarrow x =$	dM1
	x = 0.90	Al
110	CRD LD Y LD Y	
1.6	$y = \frac{c^2}{x} = c^2 x^{-1} \Rightarrow \frac{dy}{dx} = -c^2 x^{-2} \text{ or } -\frac{c^2}{x^2}$	8
	and so, at $P\left(\operatorname{ct}, \frac{c}{t}\right)$ , $m_{\tau} = -\frac{1}{t^2}$	M1
	$y - \frac{c}{c} = -\frac{1}{c}(x - ct) \Rightarrow y = -\frac{1}{c}x + \frac{2c}{c}$	M1
	$t^{2^{1/2}}$ $t^{2^{1/2}}$ $t^{2^{2^{1/2}}}$ $t^{2^{2^{1/2}}}$	A1
	$2c \left( \ldots 2c \right)$	M1
2	$y = 0 \Rightarrow x = 2ct (\Rightarrow x_A = 2ct), x = 0 \Rightarrow y = \frac{1}{t} (\Rightarrow y_A = \frac{1}{t})$	A1
	$(OB = 4OA \Rightarrow) \frac{2c}{t} = 4(2ct) \Rightarrow t = \dots$	M1
	$\left(t^2 = \frac{1}{4} \Rightarrow\right) t = \frac{1}{2} \text{ or } 0.5$	Al
	$(\text{Area OAB} = 72 \Rightarrow) \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 72 \Rightarrow c =(\Rightarrow c = 6)$	M1
	Deduces the numerical value $x_p$ and $y_p$ using their values of t and c	M1
	P(3, 12)  or  x = 3  and  y = 12	Al

# AS Further Pure Mathematics 1 Mark Scheme Vol 1 Set 24

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AS Further Mechanics 1 Vol 1 Set 33

## 1. Work, Energy and Power 7



ngole i

A small book of mass *m* is held on a rough straight desk lid which is inclined at an angle  $\alpha$  to the horizontal, where tan  $\alpha = \frac{3}{4}$ . The book is released from rest at a distance of 0.7m from the edge of the desk lid, as shown in Figure 1. The book slides down the desk lid and then hits the floor that is 0.7m below the edge of the desk lid. The coefficient of friction between the book and the desk lid is 0.2

The book is modelled as a particle which, after leaving the desk lid, is assumed to move freely under gravity.

a. Find, in terms of *m* and *g*, the magnitude of the normal reaction on the book as it slides down the desk lid.

#### (2 marks)

b. Use the work-energy principle to find the speed of the book as it hits the floor.

(5 marks)

AS Further Mechanics 1 Vol 1 Set 33

## 2. Collisions with Two Particles 4

Two particles, A and B, have masses 3m and 9m respectively. The particles are moving in opposite directions along the same straight line on a smooth horizontal plane when they collide directly.

Immediately before they collide, A is moving with speed 4u and B is moving with speed 3u.

The direction of motion of each particle is reversed by the collision.

In the collision, the magnitude of the impulse exerted on A by B is  $\frac{63mu}{2}$ 

a. Find the value of the coefficient of restitution between A and B.

## (7 marks)

b. Hence, write down the total loss in kinetic energy due to the collision, giving a reason for your answer.

(1 mark)

Question	Scheme	Mark
	Resolve perpendicular to the plane.	M1
la	$R = \frac{4}{5}mg$	A1
	Work done against friction = $0.2R \times 0.7$ (= $0.112mg$ )	M1
	PE Loss = $mg \times 0.7 \sin \alpha + 0.7 mg$ (= 1.12 $mg$ )	M1
1b	Using work-energy principle	M1
	$1.12mg = 0.112mg + \frac{1}{2}mv^2$	A1
	$v = 4.4 \text{ or } 4.44 \text{ (ms}^{-1}\text{)}$	A1
	$\begin{array}{c} 2 \\ 2 \\ \hline 2 \\ \hline 2 \\ \hline 3m \\ \hline 9m \\ \hline 9m \\ \hline 2 \\ 2 \\$	
	$ \begin{array}{c} 2 \\ 3m \\ 4u \\ 4u \\ V_A \\ V_B \end{array} $	
20	$ \begin{array}{c} 2 \\ 3m \\ 4u \\ 4u \\ V_A \end{array} $ $ \begin{array}{c} 3u \\ V_B \end{array} $ Use of Impulse-Momentum principle $ \begin{array}{c} 5m \\ 5m \\$	M1
2a	$2 \qquad 3m \qquad 9m \qquad 2$ $-4u \qquad 3u \qquad 3u \qquad 3u \qquad 4u \qquad 3u \qquad 3u \qquad 3u \qquad $	M1 A1
2a	$2 \qquad 3m \qquad 9m \qquad 2$ $-4u \qquad 3u \qquad 3u \qquad v_8  $	M1 A1 M1
2a	$2 \qquad 3m \qquad 9m \qquad 2$ $-4u \qquad 3u \qquad 3u \qquad v_{B} \qquad v_$	M1 A1 M1 A1
2a	$\frac{2}{40}$ $\frac{40}{v_{A}}$ $\frac{30}{v_{B}}$ $\frac{40}{v_{A}}$ $\frac{30}{v_{B}}$ $\frac{30}{v_{B}}$ Use of Impulse-Momentum principle For A: $\frac{63mu}{2} = 3m(v_{A} - (-4u))$ Use of Impulse-Momentum principle For B: $\frac{63mu}{2} = 9m(v_{B} - (-3u))$ $\frac{13u}{2} \text{ and } v_{B} = \frac{u}{2}$	M1 A1 M1 A1 A1
2a	$\frac{2}{40}$ $\frac{40}{\sqrt{A}}$ $\frac{30}{\sqrt{B}}$ $\frac{40}{\sqrt{A}}$ $\frac{30}{\sqrt{B}}$ Use of Impulse-Momentum principle For A: $\frac{63mu}{2} = 3m(v_A - (-4u))$ Use of Impulse-Momentum principle For B: $\frac{63mu}{2} = 9m(v_B - (-3u))$ $\frac{13u}{\sqrt{A}} = \frac{13u}{2}$ and $v_B = \frac{0}{2}$ $\frac{13u}{2} + \frac{0}{2}$ $e = \frac{13u}{2} + \frac{0}{2}$	M1 A1 M1 A1 A1 A1 M1
2a	$\frac{2}{4u} \xrightarrow{3m}} \frac{9m}{2}$ $\frac{4u}{\sqrt{x_{A}}} \xrightarrow{3u}} \xrightarrow{2}$ Use of Impulse-Momentum principle For A: $\frac{63mu}{2} = 3m(v_{A} - (-4u))$ Use of Impulse-Momentum principle For B: $\frac{63mu}{2} = 9m(v_{B} - (-3u))$ $v_{A} = \frac{13u}{2} \text{ and } v_{B} = \frac{u}{2}$ $e = \frac{13u}{4u + 3u}$ $e = 1$	M1 A1 M1 A1 A1 M1 A1 A1

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## AS Further Statistics 1 Vol 1 Set 49

## 1. Chi-Squared Test 2

In a game, a coin is spun 5 times and the number of heads obtained is recorded. Jimmy suggests playing the game 20 times and carrying out a chi-squared test to investigate whether the coin might be biased.

a. Explain why playing the game only 20 times may cause problems when carrying out the test.

(1 mark)

Stuart decides to play the game 500 times. The results are as follows

Number of heads	0	1	2	3	4	5
Observed frequency	8	35	124	163	112	58

Stuart decides to test whether or not the data can be modelled by a binomial distribution, with the probability of a head on each spin being 0.59

He calculates the expected frequencies, to 2 decimal places, as follows

Number of heads	0	1	2	3	4	5
Expected frequency	5.79	41.68	119.96	172.62	124.20	35.75

b. State the number of degrees of freedom in Stuart's test, giving a reason for your answer. (1 mark)

c. Carry out the test at the 5% level of significance. You should state your hypotheses, test statistic, critical value and conclusion clearly.

(5 marks)

d. Showing your working, find an alternative model which would better fit Stuart's data.

(2 marks)

AS Further Statistics 1 Vol 1 Set 49

## 2. Discrete Random Variables 3

The discrete random variable X has probability distribution

x	-3	-2	0	1	2
P(X = x)	q	q	r	1 10	$\frac{1}{10}$

Where q and r are probabilities.

- a. Write down, in terms of q and r,  $P(X \le 0)$
- b. Show that  $E(X^2) = \frac{1}{2} + 13q$

Given that  $E(X^3) = E(3X^2) + E(18X)$ 

- c. find the value of q and the value of r
- d. Hence find  $P(X^3 > 3X^2 + 18X)$

(1 mark)

(2 marks)

(7 marks)

(4 marks)

Question	Scheme	Marks
la	Not all the expected frequencies are likely to be over 5 Or the sample size is too small	B1
lb	5 degrees of freedom since the parameter is not estimated from the data [and the totals agree]	B1
	H <sub>0</sub> : B(5, 0.59) is a suitable model H <sub>1</sub> : B(5, 0.59) is not a suitable model	B1
	$\sum \frac{(O-E)^2}{E} = \frac{(8-5.79)^2}{5.79} + \dots + \frac{(58-35.75)^2}{35.75}$	M1
1c	= 17.6326 awrt 17.6	Al
	$\chi^2_{5, 0.05} = 11.070$	B1ft
	17.6 > 11.070 B(5, 0.59) is not a suitable model [for the number of heads spun]	Alft
14	$\frac{(0 \times 8) + (1 \times 35) + (2 \times 124) + (3 \times 163) + (4 \times 112) + (5 \times 58)}{500} [= 3.02]$	M1
id	$B\left(5, p = \frac{3.02}{5} = 0.604\right)$	Al
		· · · · ·
2a	2q + r	B1
2b	$E(X^{2}) = (-3)^{2} \times q + (-2)^{2} \times q + 0^{2} \times r + 1^{2} \times \frac{1}{10} + 2^{2} \times \frac{1}{10}$	M1
20	$=\frac{1}{2}$ + 13q from correct working	Alcso
1.66	$E(X) = -3q - 2q + \frac{1}{10} + \frac{2}{10} \left[ = \frac{3}{10} - 5q \right]$	M1
100	$E(3X^2 + 18X) = \frac{69}{10} - 51q$	Al
110.0	$E(X^{3}) = (-3)^{3} \times q + (-2)^{3} \times q + 0^{3} \times r + 1^{3} \times \frac{1}{10} + 2^{3} \times \frac{1}{10}$	M1
2c	$=\frac{9}{10}-35q$	Al
1.12	Sum of probabilities=1 gives: $2q + r = \frac{4}{5}$	M1
1.15	Solve: $16q = 6$ and $2q + r = \frac{4}{5}$	dM1
	So $r = \frac{1}{20}$ and $q = \frac{3}{8}$	Al
	$X^{3} > 3X^{2} + 18X \implies X(X - 6)(X + 3) > 0$	M1
24	Use of sketch or table to see: $-3 < X < 0$ or $X > 6$	Al
20	So $P(X^3 > 3X^2 + 18x) = P(X = -2)$	M1
	$=q=\frac{3}{8}$	Alft

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#### A Level Core Pure Mathematics Vol 1 Set 30

#### 1. Improper Integrals

Show that

$$\int_{0}^{\infty} \frac{18x - 12}{(3x^{2} + 2)(x + 1)} dx = \ln k$$

where k is a rational number to be found.

2. Polar Equations





Figure 1 shows a sketch of two curves  $C_1$  and  $C_2$  with polar equations

 $\begin{array}{ll} C_1: & r = 5(1 + \sin \theta) & 0 \le \theta < 2\pi \\ C_2: & r = 15(1 - \sin \theta) & 0 \le \theta < 2\pi \end{array}$ 

The region R lies inside  $C_1$  and outside  $C_2$  and is shown shaded in Figure 1.

Show that the area of R is

 $p\sqrt{3}-q\pi$ 

where p and q are integers to be found.

(9 marks)

(7 marks)

Question	Scheme	Mark
	$\frac{18x - 12}{(3x^2 + 2)(x + 1)} = \frac{Ax + B}{3x^2 + 2} + \frac{C}{x + 1}$	M1
	$18x - 12 = (Ax + B)(x + 1) + C(3x^{2} + 2)$ $x = -1 \Rightarrow C = -6, x = 0 \Rightarrow B = 0, x = 1 \Rightarrow A = 18$ Or Compares coefficients and solves $(A + 3C = 0, A + B = 18, B + 2C = -12)$ $\Rightarrow A =, B =, C =$	dM1
	A = 18, B = 0, C = -6	A1
1	$\int \left(\frac{18x}{3x^2+2} - \frac{6}{x+1}\right) dx = 3\ln(3x^2+2) - 6\ln(x+1)$	A1ft
	$3\ln(3x^{2}+2) - 6\ln(x+1) = \ln\left(\frac{(3x^{2}+2)^{3}}{(x+1)^{6}}\right)$	M1
	$\lim_{x \to \infty} \left( \ln \frac{(3x^2 + 2)^3}{(x + 1)^6} \right) = \ln 8$	B1
-12	$\Rightarrow \int_{0}^{\infty} \frac{18x - 12}{(3x^{2} + 2)(x + 1)} dx = \ln \frac{8}{27} \operatorname{cao}$	A1
- 63	NA CONCRETE AND A CONCRETE	1.1
1.6.6	$5(1 + \sin\theta) = 15(1 - \sin\theta) \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{\pi}{6} \left( \text{or } \frac{5\pi}{6} \right)$	A1
	Use of $\frac{1}{2}\int (5(1+\sin\theta))^2 d\theta$ or $\frac{1}{2}\int (15(1-\sin\theta))^2 d\theta$	M1
2	$\left(\frac{1}{2}\right)\int \left[25(1+\sin\theta)^2 - 225(1-\sin\theta)^2\right]d\theta$ $= \left(\frac{1}{2}\right)\int \left[25+50\sin\theta + 25\sin^2\theta - 225+450\sin\theta - 225\sin^2\theta\right]d\theta$ or $\int 25(1+\sin\theta)^2d\theta = \int \left(25+50\sin\theta + 25\sin^2\theta\right)d\theta$ and $\int 225(1-\sin\theta)^2d\theta = \int \left(225-450\sin\theta + 225\sin^2\theta\right)d\theta$	M1 A1
	$\int \sin^2 \theta  d\theta = \frac{1}{2} \int (1 - \cos 2\theta) d\theta \Rightarrow$ $\int \left[ 25(1 + \sin \theta)^2 - 225(1 - \sin \theta)^2 \right] d\theta = 200 \sin 2\theta - 500 \cos \theta - 300\theta$	M1 A1
	$A = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left[ 25(1 + \sin\theta)^2 - 225(1 - \sin\theta)^2 \right] d\theta$	dM1
	$\frac{-2[(-100\sqrt{3} + 250\sqrt{3} - 250\pi) - (100\sqrt{3} - 250\sqrt{3} - 50\pi)] = \dots}{-2}$	
	$= 150\sqrt{3} - 100\pi$	A1

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A Level Further Mechanics 1 Vol 1 Set 51

## 1. Oblique Impacts 2



Figure 1 represents the plan view of part of a horizontal floor, where AB and CD represent fixed vertical walls, with AB parallel to CD.

A small ball is projected along the floor towards wall AB. Immediately before hitting wall AB, the ball is moving with speed v ms<sup>-1</sup> at an angle  $\alpha$  to AB, where  $0 < \alpha < \frac{\pi}{2}$ 

The ball hits wall AB and then hits wall CD.

After the impact with wall CD, the ball is moving at angle  $\frac{1}{2}\alpha$ 

The coefficient of restitution between the ball and wall AB is  $\frac{12}{73}$ 

The coefficient of restitution between the ball and wall CD is also  $\frac{12}{73}$ 

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

a. Show that  $\tan\left(\frac{1}{2}\alpha\right) = \frac{71}{73}$ 

#### (7 marks)

a. Find the percentage of the initial kinetic energy of the ball that is lost as a result of the two impacts.

(4 marks)

A Level Further Mechanics 1 Vol 1 Set 51

## 2. Elastic Springs 2



A light elastic spring has natural length 61 and modulus of elasticity 9mg.

One end of the spring is attached to a fixed point X on a rough inclined plane.

The other end of the spring is attached to a package P of mass m.

The plane is inclined to the horizontal at an angle  $\alpha$  where tan  $\alpha = \frac{7}{24}$ 

The package is initially held at the point Y on the plane, where XY = 2I. The point Y is higher than X and XY is a line of greatest slope of the plane, as shown in Figure 2.

The package is released from rest at Y and moves up the plane.

The coefficient of friction between P and the plane is  $\frac{1}{2}$ 

By modelling P as a particle,

a. show that the acceleration of P at the instant when P is released from rest is  $\frac{27}{5}g$ 

## (5 marks)

a. find, in terms of g and l, the speed of P at the instant when the spring first reaches its natural length of 6l.

(6 marks)

Question	Scheme	Marks
	Use model to find components of velocity after the impacts:	
	$\int \frac{12}{73} v \sin \alpha$	B1
	$\vee \cos \alpha$	B1
la	$\vee \cos \alpha$	B1
	$\sqrt{\frac{144}{5329}}$ v sin $\alpha$	B1
	$\tan\frac{\alpha}{2} = \frac{\frac{144}{5329} \text{v} \sin \alpha}{\text{v} \cos \alpha} \left( = \frac{144}{5329} \tan \alpha \right)$	М1
	$t = \tan \frac{\alpha}{2} \Rightarrow t = \frac{144 \times 2t}{5329(1-t^2)}$	M1
- 19	$1 - t^2 = \frac{288}{5329} \Rightarrow t^2 = \frac{5041}{5329} \Rightarrow \tan\frac{\alpha}{2} = \frac{71}{73}$	Al
	$\tan \alpha = \frac{\frac{142}{73}}{1 - \frac{5041}{5329}} = \frac{5183}{144}$	B1
1b	change in Kinetic Energy : $\frac{1}{2}mv^2 - \frac{1}{2}m\left(v^2\cos^2\alpha + \left(\frac{144}{5329}v\right)^2\sin^2\alpha\right)$	М1
	% of Kinetic Energy lost = $100 \left( \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv^2 \left( \left(\frac{144}{5185}\right)^2 + \left(\frac{144}{5329}\right)^2 \times \left(\frac{5183}{5185}\right)^2 \right)}{\frac{1}{2}mv^2} \right)$	MI
	99.84 (%)	Al

# A Level Further Mechanics 1 Mark Scheme Vol 1 Set 51

Question	Scheme		Marks
2a	Thrust in the spring = $\frac{9mg \times 4l}{6l}$ (= 6mg)		B1
	Equation of motion:		M1
	$6mg - mgsin \alpha - \frac{1}{3}mgcos \alpha = ma$	Unsimplified equation with at most one error	Alft
	$\left(6mg - \frac{7mg}{25} - \frac{8mg}{25} = ma\right)$	Correct unsimplified equation	Alft
	$a = \frac{27g}{5}$ from correct working		Al
	Initial EPE = $\frac{9mg \times 16l^2}{2 \times 6l}$ (= 12mgl)		B1
	Gain in GPE = $mg \times 4lsin \alpha \left(=\frac{28}{25}mgl\right)$		B1
2b	Work done against friction = $\frac{1}{3}$ mgcos $\alpha \times 4l \left(=\frac{32}{25}$ mgl $\right)$		B1
	Work-energy equation:		M1
	$\frac{1}{2}mv^2 + \frac{32}{25}mgl + \frac{28}{25}mgl = 12mgl$		A1
1.13	$v = \sqrt{\frac{96gl}{5}}$		A1

Abbreviations			
м	Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated		
А	Accuracy marks can only be awarded if the relevant method (M) marks have been earned		
В	Unconditional accuracy marks (independent of M marks)		
ft	follow through		
CSO	correct solution only. There must be no errors in this part of the question to obtain this mark		
oe	or equivalent (and appropriate)		