Curves 2

a. Factorise completely $4x - x^3$

(2 marks)

The curve *C* has equation

$$y = 4x - x^3$$

b. Sketch *C* showing the coordinates of the points at which the curve cuts the x-axis.

(2 marks)

The line *l* has equation *y* = *k* where *k* is a constant.

Given that *C* and *l* intersect at 3 distinct points,

c. find the range of values for *k*, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable

(3 marks)

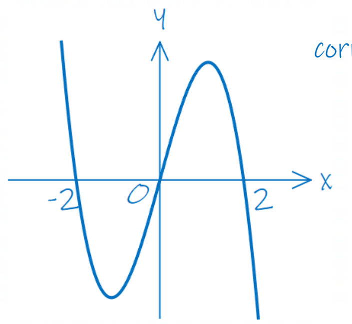
a. $4x - x^3 = x(4 - x^2)$ 1 mark

$= x(2 - x)(2 + x)$ 1 mark

Using the difference of two squares

b.

the values where the curve cuts the x-axis are the solutions of the equation $4x - x^3 = 0$



correct orientation of the cubic graph 1 mark

curve passing through -2, 0 and 2 1 mark

c. The line *l* is parallel to the *x*-axis. For it to cut through *C* at 3 distinct points, it will need to be below the maximum turning point and above the minimum turning point.

Solve $\frac{dy}{dx} = 0$ to find the *x*-coordinates of the turning points.

$$y = 4x - x^3 \Rightarrow \frac{dy}{dx} = 4 - 3x^2$$

$$4 - 3x^2 = 0$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \pm\sqrt{\frac{4}{3}}$$

$$x = \pm\frac{2\sqrt{3}}{3}$$

1 mark

Substitute these values into $y = 4x - x^3$ to get the corresponding *y*-coordinates

$$y = 4\left(\pm\frac{2\sqrt{3}}{3}\right) - \left(\pm\frac{2\sqrt{3}}{3}\right)^3$$

$$= \pm\frac{8\sqrt{3}}{3} \mp \frac{8\sqrt{3}}{9}$$

$$= \pm\frac{16\sqrt{3}}{9}$$

1 mark

So, using set notation: $\left\{k \in \mathbb{R} : -\frac{16\sqrt{3}}{9} < k < \frac{16\sqrt{3}}{9}\right\}$ 1 mark